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OPTIMAL DESIGN OF INDUSTRIAL MANIPULATOR TRAJECTORY FOR MINIMAL TIME OPERATION

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The operation time of an industrial manipulator which moves along the arbitrary trajectory form initial to final position is minimized. The Hartenberg-Denavit coordinates and its transformations are employed to express the motion of manipulator joints and the method of finding inverse kinematic solution is established to minimize the operation time. The dynamic equations for a manipulator of n joints are transformed into the equation of 1 D.O.F. by utilizing path coordinates. By using a recently developed method, the allowable maximum speed limit curve is obtained from the torque limits of joint driver, and the curve is used to determine the optimal velocity curve which minimizes the operation time. The minimum time trajectory planning is applied to an industrial manipulator resulting in the performance improvement of the manipulator. As a demonstrative example, we have simulated the proposed algorithm with a dynamic model of PUMA 560 on an IBM PC.

Key Words: Time Optimal Phase Plane Trajectory, Minimal Time Operation, Dynamics of Industrial Manipulator, Allowable Maximum Velocity Limit Curve, Switching Point

1. INTRODUCTION

The demand for productivity improvement in these days requires the industrial automation, for which robot manipulators have emerged as a major manufacturing components. High productivity of robot manipulators performing tasks such as material handling, welding, spray painting can be achieved by having manipulators move more quickly. However, manipulator has a limited capability in following the given path and generally the actuator's torque may be arbitrary function of the kinematic and dynamic characteristics of the manipulators. Therefore most of industrial manipulators have been programmed to move along their paths with constant accelerations or velocities. These velocities and accelerations are usually selected not to exceed their maximum values at any point along the path so as the manipulator motion not to leave its path. Practically the actuator torques have near maximum value at only a few points on the path and their operation performance will be less than their capacity along the path. A manipulator could operate faster by utilizing its full capacity at every point along the path.

Considerable research has been done to find out analytical solutions for improving productivity by minimizing operation time. Kahn and Roth(1971) presented time-optimum trajectory planning problem for a manipulator traveling between two end points in which the path was not specified. The maximum forces and torques were used as constraints by assuming them to be constant. A simplified linearized manipulator model resulted in bang-bang control. Lin and Chang(1983) carried out the formulation and optimization of cubic polynomial trajectory for manipulator with constraints on the velocity, acceleration and jerk of joint which were expressed in joint coordinates system. When a new trajectory is given, Hollerbach(1984) obtained an equation for a new trajectory by simply linearizing the original dynamic equation. It is however, difficult to apply his method to the operation at a high speed or rapid speed change.

Ailon and Langholtz(1985) mathematically proved the existence of time optimal control. Rajan(1985) found the minimum time trajectory for the 2 D.O.F. manipulator for specified path using spline when the motions of acurators are constrained. He also suggested the method of improving the minimum time by changing the path. Sahar(1985) altered the working space into a type of mosaic by using grid points and selected the grid points by applying time scale algorithm to find the linearized optimal trajectory. However, a real optimal trajectory is nonlinear and it is difficult to obtain an exact solution with this method since the nonlinearity is neglected. Shin and Mckay(1985) have attempted to develop approximate solutions to the problem to reduce the computational effort. In their works, the problem is limited to finding the manipulator's velocity profile along a prescribed path so that it is traversed in minimum time without violating the physical capabilities such as maximum velocity or torque. Kim and Shin(1985) performed optimal control with a weighted time operation cost using a concept of averaged dynamics for each sampling interval on a path. Shiller and Dubowsky(1985) analytically calculated the optimal time according to a prescribed path with the constraints on actuator and the payload at end effector was included in their calculations. Norris et. al(1986) found the path taken less time by utilizing the minimum time planning for a specified path and the parameterized Bezier spline when the constraints are given on the actuator and its path. Pfeiffer and Johanni(1987) solved Lagrange-Euler dynamic equation of n D.O.F. manipulator by reducing the equation into 1 D.O.F. problem through the path coordinates. They provided the method of reducing the opera-

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tional cost and the jerk effect by considering the weighted minimal time operation cost and driving torque.

In this work, the operation time of an industrial manipulator which moves along arbitrary trajectory from initial to final position is minimized. The Hartenberg-Denavit coordinates and its transformations are employed to express the motion of manipulator joints, and the method of finding the inverse kinematic solution is established to minimize the operation time. The dynamic equations for manipulator are transformed into the equation of 1 D.O.F. by utilizing path coordinates. By using a recently developed method the allowable maximum speed limit curve is obtained from the torque limits of joint actuators, and the curve is used in determining the optimal velocity curve which minimize the operation time. The minimum time trajectory planning is applied to an industrial manipulator to prove the productivity improvement of the manipulator.

2. COORDINATES SYSTEM AND HOMOGENEOUS TRANSFORMATION

There are two coordinates system used to describe the end effector position of a manipulator ; (1) reference(absolute) coordinates system. (2) joint(relative) coordinates system. The reference coordinates system is a right hand rectangular coordinates system fixed to manipuiator base, where as the joint coordinates system is the system expressing the relative position between joints. The joint coordinates system is measured in radian for a revolute joint or in length unit for a prismatic joint.

The joint coordinates system, which is convenient to use, was suggested by Hartenberg and Denavit(1955) in the motion analysis of passive mechanism i.e. linkages. Instead of using three translational and 3 rotational parameters to define the spatial motion, they simply described the motion by introducing 4 parameters of a, α, d, θ as shown in Fig. 1. These 4 patameters are usually called kinematic parameters of a robot manipulator.

In the real motion of manipulator only one among 4 parameters becomes variable ; θ for revolute joint, d for prismatic joint and the remainder becomes constant. The relative positions between joints and orientation/position of the end effector are obtained through 4×4 homogeneous transformation matrix as given in Eq.(1)

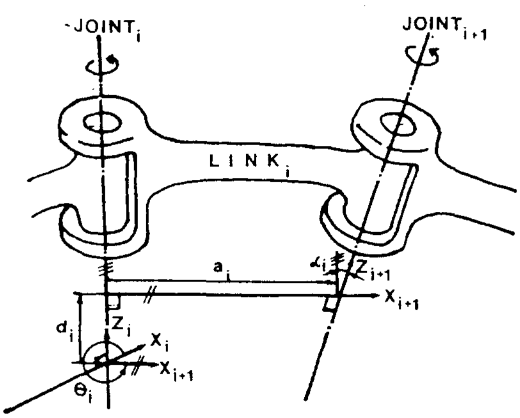

Fig. 1 Link parameters

$$A_i=\left[\begin{array}{c|c} R(\text{rotation}) & P(\text{translation}) \\ \hline 0 & 1 \end{array}\right] \tag{1}$$

$$=\begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. FORWARD KINEMATICS AND INVERSE KINEMATICS

Forward kinematics for a manipulator is the process to calculate the position (or motion) of an end effector when joint variables are specified. The process is carried out by multiplying sequentially the 4×4 homogeneous transformation matrix of each joint as in Eq.(2) and finding the transformation matirx T_i for the corresponding position(or motion).

$T_i=A_1\ A_2\ \cdots\cdots\ A_i$ when A_i is the transformation matrix (2)

In contrast to forward kinematics, the inverse kinematics is employed to find the joint variables inversely from the position(or motion) of end effector. The process of the inverse kinematics is briefly discussed below.

Assume that the end effector is located at j th joint and its homogeneous transformation matrix is given. Then the joint variable q in general coordinates system is determined sequentially from q_1 to q_j.

Premultiplying A_1^{-1} to $T_j=A_1\ A_2\ \cdots\cdots\ A_j$ gives

$$A_1^{-1}T_j=A_2\ A_3\ \cdots\cdots\ A_j \tag{3}$$

Successively premultiplying Eq.(3) by the A matrix inverses gives

$$\begin{aligned} &A_2^{-1}A_1^{-1}T_j=A_3\ \cdots\cdots\ A_j \\ &A_{j-1}^{-1}A_{j-2}^{-1}\ \cdots\cdots\ A_2^{-1}A_1^{-1}T_j=A_j \end{aligned} \tag{4}$$

Eq.(3) and Eq.(4) give $j-1$ equations which consist of known T_j and joint variables θ_i on its left hand side and zeros, constant or joint variables on the right side. From Eq.(3), 12 equations are obtained and the first variable q_1 is found by comparing the terms on both sides. The other variables q_2 $\cdots\cdots$ q_j are sequentially determined by substituting q_1 into Eq.(4) and repeating this procedure.

4. KINEMATIC DIFFERENTIAL RELATIONSHIPS

The differential relationships between the reference coordinates system for the end effector position and the joint coordinates system is necessary for the calculation of Lagrange-Euler equations that gives the dynamic equations for the industrial manipulator.

Let q_i be the i th joint variable of the manipulator and dA_i be a change of A_i for the infinitesimal change of q_i. Then A_i+dA_i is equal to the infinitesimal rotation $d\theta$ of A_i with respect to vector $\boldsymbol{k}$ plus infinitesimal translations dx, dy, dz, This can be expressed as

$$A_i + dA_i = \text{Trans}(dx, dy, dz)\ \text{Rot}(\boldsymbol{k}, d\theta) A_i \tag{5}$$

Accordingly

$$dA_i = \{\text{Trans}(dx, dy, dz)\ \text{Rot}(\boldsymbol{k}, d\theta) - \mathrm{I}\} \cdot A_i \tag{6}$$

Let θ_i be the differential operator and then Eq.(6) becomes

$$dA_i = Q_i A_i \tag{7}$$

where

$$Q_i = \{\text{Trans}(dx, dy, dz)\ \text{Rot}(\boldsymbol{k}, d\theta) - \mathrm{I}\} = \begin{bmatrix} 0 & -k_z d\theta & k_y d\theta & dx \\ k_z d\theta & 0 & -k_y d\theta & dy \\ -k_y d\theta & k_y d\theta & 0 & dz \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{8}$$

In Eq.(8)

$k_x = k_y = dx = dy = dz = 0$ for revolute joints
$k_x = k_y = k_z = dx = dy = 0$ for prismatic joints

The differential change of $T_j (j >= 1)$ with repect to q_i becomes

$$\frac{\partial T_j}{\partial q_i} = \frac{\partial}{\partial q_1}(A_1\ A_2 \cdots\cdots A_{i-1}\ A_i \cdots\cdots A_j) = (A_1 \cdots A_{i-1}) Q_i (A_1 \cdots A_{i-1})^{-1} T_j \tag{9}$$

where

$${}^j\Delta_i = (A_1 \cdots A_{i-1}) Q_i (A_1 \cdots A_{i-1})^{-1}$$

Since

$$(A_i \cdots A_{i-1}) = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{10}$$

${}^j\Delta_i$ can be expressed in a matrix form

$${}^j\Delta_i = \begin{bmatrix} 0 & -a_z & o_z & (\boldsymbol{p}\cdot\boldsymbol{n})_z \\ a_z & 0 & -n_z & (\boldsymbol{p}\cdot\boldsymbol{o})_z \\ -o_z & n_z & 0 & (\boldsymbol{p}\cdot\boldsymbol{a})_z \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{11}$$

The differential change of $\frac{\partial T_j}{\partial q_i}$ with respect to the other variable q_k will be

$$\frac{\partial^2 T_j}{\partial q_i \partial q_k} = {}^j\Delta_k\ {}^j\Delta_i\ T_j \tag{12}$$

Eq.(9) is the component of jacobian matrix J(q) and Eq.(12) is that of Hessian matrix H(q).

Therefore, the velocity of i th link is

$$\boldsymbol{V}_i = J(\boldsymbol{q})\dot{\boldsymbol{q}} \tag{13}$$

and its acceleration becomes

$$\dot{\boldsymbol{V}}_i = J(\boldsymbol{q})\ddot{\boldsymbol{q}} + \dot{J}(\boldsymbol{q})\dot{\boldsymbol{q}} \tag{14}$$

5. DYNAMIC EQUATION OF AN INDUSTRIAL MANIPULATOR

Mechanisms are generally classified into two kinds:

(1) passive mechanisms consisting of closed loop such as 1 D.O.F. linkages

(2) active mechanisms consisting of open loop such as robots or industrial manipulators of multi D.O.F.

The rigorous analysis of the manipulator motion is extremely difficult because the driving units at joints are composed of passive mechaisms such as gear trains and D.C. servo motors. Therefore, the motion analysis of a manipulator is performed by considering the manipulator as an active mechanism.

The equation of motion of an active mechanism as the model of a manipulator can be derived by employing Lagrangian dynamics. The Lagrangian dynamics enables one to derive in a simple way the dynamic equation of motion for a complex multi D.O.F. system.

The Lagrangian equation for a manipulator with n D.O.F. is given by

$$\begin{aligned} L &= K - P \\ &= \left\{\frac{1}{2}\sum_{i=1}^{n}\text{Trace}\left(\sum_{m=1}^{i}\sum_{l=1}^{i}\frac{\partial T_i}{\partial q_m}\cdot I_i \frac{\partial T_i}{\partial q_l}\dot{q}_m\dot{q}_l\right) + \frac{1}{2}\sum_{i=1}^{n} I_i^a \dot{q}_i^2\right\} \\ &\quad - \left\{-\sum_{i=1}^{n} m_i \boldsymbol{g}(T_1\cdot\boldsymbol{r}_i)\right\} \end{aligned} \tag{15}$$

where

K = total kinetic energy of the robot manipulator
P = total potential energy of the robot manipulator
$\boldsymbol{r}_i$ = point on ith link
I_i = inertia matrix
I_i^a = inertia of an actuator
m_i = mass
$\boldsymbol{g}$ = acceleration of gravity
q_i = ith joint variable
T_i = ith transformation matrix

Applying the Lagrange-Euler formulation to the Eq.(15) yields the necessary generalized torque F_i for the actuator of joint i to drive the ith link of the manipulator.

$$\begin{aligned} F_i &= \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_1} \\ &= \sum_{j=1}^{n} D_{ij}\ddot{q}_j + I_i^a \ddot{q}_1 + \sum_{j=1}^{n}\sum_{k=1}^{n} D_{ijk}\dot{q}_j\dot{q}_k + \dot{D}_i \end{aligned} \tag{16}$$

In Eq.(16)

$D_{ii}\ddot{q}_i$ = effective inertia force of joint i, i.e. the torque at joint i caused by the acceleration of joint i
$D_{ij}\ddot{q}_i$ or $D_{ij}q_j$ = coupling inertia between joints i and j
$D_{ijj}\dot{q}_j^2$ = centrifugal force of joint i due to the velocity of joint j
$D_{ijk}\dot{q}_j\dot{q}_k + D_{ikj}\dot{q}_k\dot{q}_j$ = Coriolis force actiong at joint i due to the velocities at joints j and k
D_i = gravity force at joint i

The inertia and the gravity terms are particularly important in manipulator control since they affect the servo stability and the positioning accuracy.

The centrifugal and Coriolis forces are considered important only when the manipulator moves at high speed. However, the errors caused by these forces are relatively small.

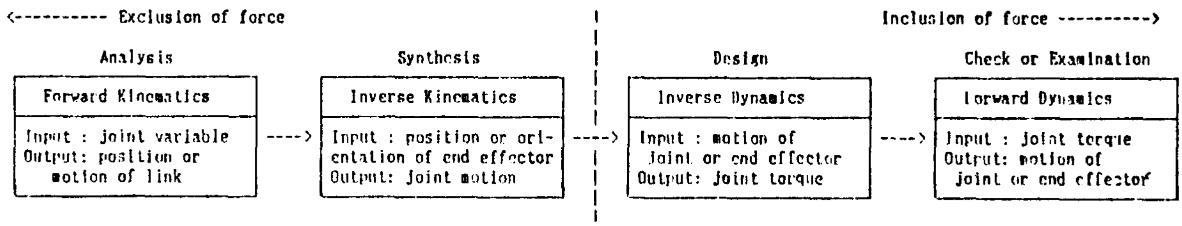

Fig. 2 Process of manlpulator dynamics

The actuator inertia I_i^a vary in magnitude according to the manipulator configuration. Those are relatively large, and have the effects of decreasing the structural dependence of the effective inertias as well as decreasing the relative importance of the coupling inertia terms. In other words, when the acurator inertia is large, it is not sufficient to consider only the manipulator structure and the inertia force should be taken into account together with the coupling inertia force.

The process to hand the dynamics of an industrial manipulator of robot can be summarized in Fig. 2.

6. OPTIMIZATION FOR MINIMAL TIME OPERATION

There are generally three methods to represent the trajectory of an end effector.

(1) Straight lines connected by circular arcs

(2) Perturbations about a straight line by using Fourier series

(3) Spline

In this work, splines are chosen to represent the path since the path curve can be adjusted conveniently by a finite set of scalar parameters. The spline function is expressed as

$$\boldsymbol{p}(u) = \boldsymbol{U}(M_b)\boldsymbol{R} \tag{17}$$

where

$$\boldsymbol{U} = (1, u, u^2, u^3),\ \boldsymbol{R}^T = (r_o, r_1, r_2, r_3)$$

$$[M_b] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix}$$

and

$\boldsymbol{r}_0 \sim \boldsymbol{r}_3$: vectors of control points

u : parameter having a value between 0 and 1, i.e. the normalized distance corresponding to the path s.

Accordingly,

$$\frac{\partial \boldsymbol{p}(u)}{\partial s} = \boldsymbol{p}_s = \boldsymbol{U}_s(M_b)\boldsymbol{R} \tag{18}$$

where

$$\boldsymbol{U}_s = \frac{\partial \boldsymbol{U}}{\partial u}\ \frac{\partial u}{\partial s}$$

Assuming the force due to the actuator inertia to be negligible, Eq.(16) can be rewritten as

$$[M]\ddot{\boldsymbol{\theta}} + \dot{\boldsymbol{\theta}}^T[C]\dot{\boldsymbol{\theta}} + \boldsymbol{G} = \boldsymbol{F} \tag{19}$$

where

$[M] = \sum D_{ij}$: inertia matrix

$[C] = \sum_{j=1}^{n}\sum_{k=1}^{n} D_{ijk}$: Coriolis, centrifugal tensor

$\boldsymbol{G} = D_i$: gravitational vector

$\boldsymbol{F} = F_i$: external force vector at actuators

Converting Eq.(19) into path coordinates(s) through kinematic transformation gives

$$\boldsymbol{p}(s) = [R]\boldsymbol{\theta}\ \ \boldsymbol{\theta} = [R]^{-1}\boldsymbol{p}(s)\ \ \dot{\boldsymbol{\theta}} = [R]^{-1}_{\theta}\boldsymbol{p}_s\dot{s} \tag{20}$$
$$\ddot{\boldsymbol{\theta}} = [R]^{-1}_{\theta}\{\boldsymbol{p}_s\ddot{s} + \boldsymbol{p}_{ss}\dot{s}^2 - ([R]^{-1}_{\theta}\boldsymbol{p}_s)^T[R]_{\theta\theta}([R]^{-1}_{\theta}\boldsymbol{p}_s)\dot{s}^2\}$$

in which

$[R]_\theta$ = Jacobian matrix $\quad [R]_{\theta\theta}$ = Hessian matrix

Substituting Eq.(20) into Eq.(19) gives

$$\boldsymbol{m}(s)\ddot{s} + \boldsymbol{b}(s)\dot{s}^2 + \boldsymbol{g}(s) = \boldsymbol{F} \tag{21}$$

where

$$\boldsymbol{m}(s) = [M][R]^{-1}_{\theta}\boldsymbol{p}_s$$
$$\boldsymbol{b}(s) = [M][R]^{-1}_{\theta}\{\boldsymbol{p}_{ss} - ([\boldsymbol{R}]^{-1}_{\theta}\boldsymbol{p}_s)^T[R]_{\theta\theta}([R]^{-1}_{\theta}\boldsymbol{p}_s)\} + ([R]^{-1}_{\theta}\boldsymbol{p}_s)^T[C][R]^{-1}_{\theta}\boldsymbol{p}_s)$$
$$\boldsymbol{g}(s) = \boldsymbol{G}$$

Eq.(21), which is a second order differential equation, can be converted to a set of 1st order differential equations and the converted equations are solved by using an integrating factor. When the initial conditions of position and velocity are s_o and $\dot{s}_o$ respectively, the linear velocity of an end effector s becomes

$$\dot{s}^2(s) = \left[\dot{s}^2(s_o) + \int_{s_0}^{s} e^{\int_{s\delta}\frac{2b}{m}ds} \cdot \frac{2(\boldsymbol{F} - \boldsymbol{g}}{\boldsymbol{m}}ds\right] e^{-\int^{s\delta}\frac{2b}{m}ds} \tag{22}$$

Hence, from Eq.(21) the maximum and minimum acceleration of ith link becomes

$$\ddot{s}_{a_i} = (\boldsymbol{F}_{imax} - \boldsymbol{b}_i\dot{s}^2 - \boldsymbol{G})/\boldsymbol{m}_i \tag{23}$$
$$\ddot{s}_{d_i} = (\boldsymbol{F}_{imin} - \boldsymbol{b}_i\dot{s}^2 - \boldsymbol{G})/\boldsymbol{m}_i$$

When s and $\dot{s}$ are given, the allowable range of acceleration can be obtained from Eq.(23)

$$\ddot{s}_{d_i} \leq \ddot{s}_s \leq \ddot{s}_{a_i}$$

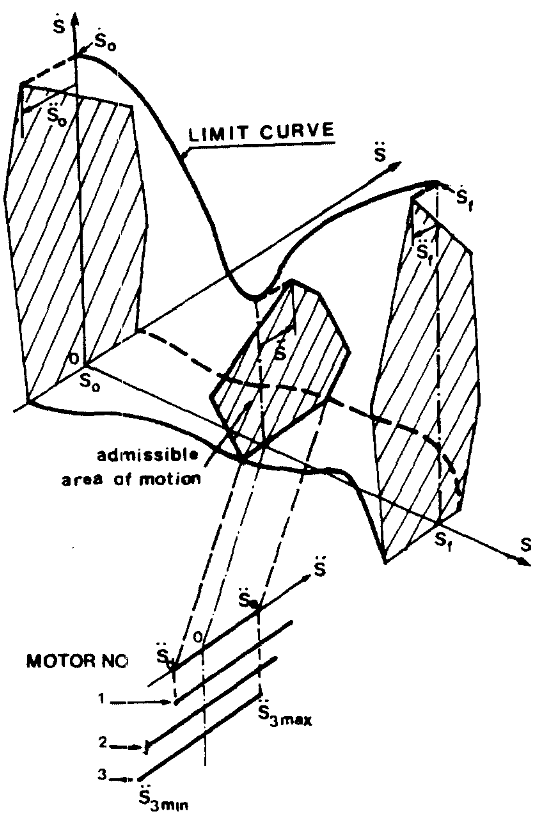

Fig. 3 Permitted acceleration range in the phase space

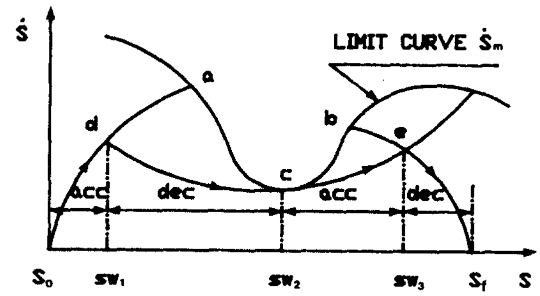

Fig. 4 Velocity limit curve and optimal velocity curve in the phase plane

The allowable range of $\ddot{s}_i$ in phase space for state variables s and $\dot{s}$ is shown in Fig. 3. From Fig. 3 and Eq.(23), the velocity at the time when the magnitude of range is zero(that is $\ddot{s}_a = \ddot{s}_d$) is the allowable maximum velocity $\dot{s}_m$ and if $\dot{s}$ is greater than $\dot{s}_m$ the end effector leaves its path.

The velocity limit curve $\dot{s}_m$ corresponding to the path s is shown in Fig. 4. In Fig. 4, the curve of s_o, d, c, e, s is the optimal velocity curve for the end effector's path and the points d, c, e are the switching points between acceleration and deceleration, which is found by forward integration for acceleration range and backward integration for deceleration range. The problem is how to find intermediate switching points like c.

The time optimal phase plane trajectory must meet the maximum velocity curve tangentially at the switching point(Yang, 1988).

In the velocity of tangent point, the j th actuator is saturated with the acceleration.

$$\ddot{s}_j = (\boldsymbol{F}_{jmin} - \boldsymbol{b}_j \dot{s}^2 - \boldsymbol{G}_j)/\boldsymbol{m}_j$$

Then there must be at least one other actuator torque $\boldsymbol{F}_k$ which, at the tangent point, satisfies the following condition.

$$\boldsymbol{F}_k = \boldsymbol{m}_k(s)\,\ddot{s} + \boldsymbol{b}_k(s)\,\dot{s}^2 + \boldsymbol{g}_k(s) \qquad (24)$$

The simplified expression for the necessary condition of tangent point is given by

$$A_1(s)\,\dot{s}^2 + A_2(s) = 0 \qquad (25)$$

where

$$A_1(s) = \frac{dB_1}{ds} - 2B_1\frac{\boldsymbol{b}_j(s)}{\boldsymbol{m}_j(s)} \qquad A_2(s) = \frac{dB_2}{ds}2B_1\frac{\boldsymbol{G}_j(s)}{\boldsymbol{m}_j(s)}$$

and

$$B_1(s) = \boldsymbol{b}_k(s) - \boldsymbol{b}_j(s)\,\boldsymbol{m}_k(s)/\boldsymbol{m}_j(s)$$
$$B_2(s) = \boldsymbol{G}_k(s) - \boldsymbol{G}_j(s)\,\boldsymbol{m}_k(s)/\boldsymbol{m}_j(s)$$

Therefore, for each value of $\dot{s}_m$, Eq.(25) is solved for s which satisfies the condition,

$$\boldsymbol{F}_k = \boldsymbol{F}_{kmax}(\text{if } \boldsymbol{m}_k > 0)$$
$$\boldsymbol{F}_k = \boldsymbol{F}_{kmin}(\text{if } \boldsymbol{m}_k < 0)$$

If the above condition is satisfied, $(s^*,\ \dot{s}^*)$ can be a tangent point.

With the optimal trajectory determined in this way, the values of $\dot{\theta}_i$, $\ddot{\theta}_i$ and $\boldsymbol{F}_i$ can be found from Eq.(20) and Eq.(21). The time taken from initial position s_o to final position s_f along the optimal trajectory becomes

$$t(s) = \int_{s_o}^{s_f} \frac{1}{\dot{s}(s)} ds \qquad (26)$$

7. EXAMPLE

The minimum time trajectory planning described above is applied to PUMA 560 (Fig.5) treating the manipulator as 3R active mechanism. The kinematic parameters (a, α, θ, d) of PUMA 560 are given in Table 1 and its general data in Table 2.

Two end points and control points of the trajectory are chosen as follows

$$\boldsymbol{r}_0 = (0.4,\ -0.4,\ -0.1)$$
$$\boldsymbol{r}_1 = (0.5,\ -0.05,\ 0.04)$$
$$\boldsymbol{r}_2 = (0.5,\ 0.2,\ 0.5)$$
$$\boldsymbol{r}_3 = (0.4,\ 0.4,\ 0.3)$$

The joint angles along the path and their time derivatives are shown in Fig. 6 and Fig. 7 respectively. Fig. 8 shows the velocity limit curve and optimal velocity curve along the path, where the curves are compared with conventional velocity control(constant velocity and acceleration). The total length of trajectory is 1064 mm, the operation time taken along the optimal trajectory is calculated as 0.419 sec.

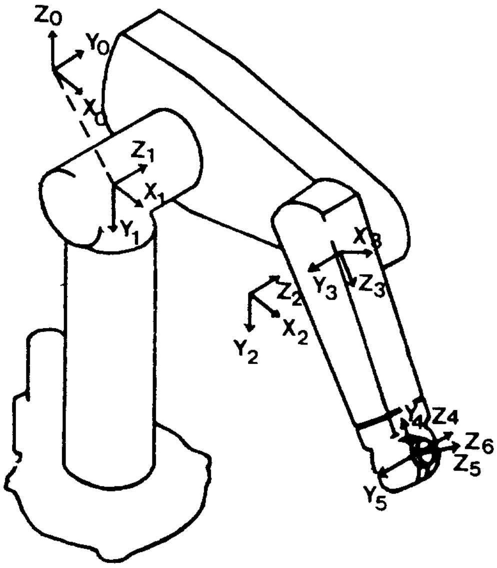

Fig. 5 Frame assignments for the PUMA 560 manipulator

Table 1 Geometric parameters for PUMA 560

JOINT	θ	α	a	d
1	θ_1	$-90°$	0	
2	θ_2	$0°$	a_2	0
3	θ_3	$90°$	a_3	d_3
4	θ_4	$-90°$	0	d_4
5	θ_5	$90°$	0	0
6	θ_6	$0°$	0	0

$a_2=0.432$, $a_3=-0.2$, $d_3=0.149$, $d_4=0.433$

Table 2 General data for PUMA 560

Link	p	1	2	3	unit
Mass	M_p	2.27	15.91	11.36	kg
Center	x	0.00	0.00	7.50	cm
of	y	−21.60	0.00	0.00	
Mass	z	0.00	0.00	21.60	
Radius	K_x	4.08	4.08	8.63	cm
of	K_y	24.94	8.82	8.82	
Gyration	K_z	2.55	2.55	24.94	
Torque	max	40.0	130.0	30.0	Nm
	min	−40.0	−130.0	−30.0	
Theta	max	160.0	45.0	225.0	deg
	min	−160.0	−225.0	−45.0	

and the time taken for conventional control is calculated as 0.542 sec. Thus the minimum time trajectory planning reduced the operation time as much as 23%.

The position, velocity and acceleration of the end effector along the path are shown in Fig. 9. And Fig. 10 shows the torque curves of 1st, 2nd and 3rd actuators for optimal trajectory operation. From Fig. 10, it is recognized that the torque curves satisfy well the given allowable range which are

$$-40<F_1<40$$
$$-130<F_2<130$$
$$-30<F_3<30$$

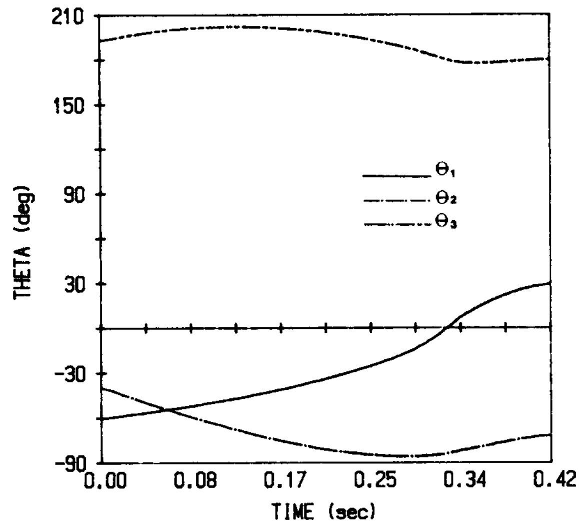

Fig. 6 Joint angles along path

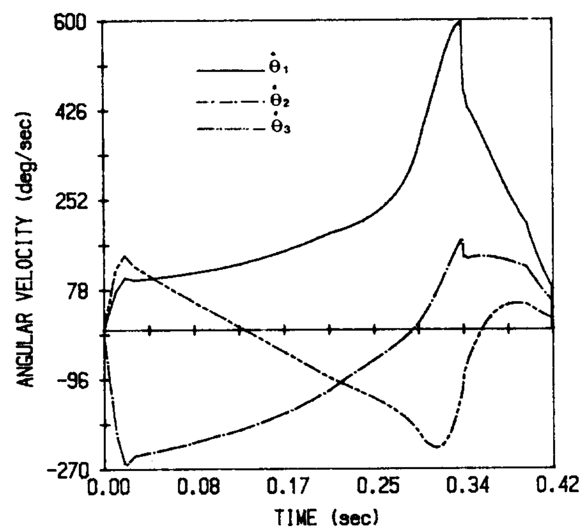

Fig. 7 Joint angular velocities along path

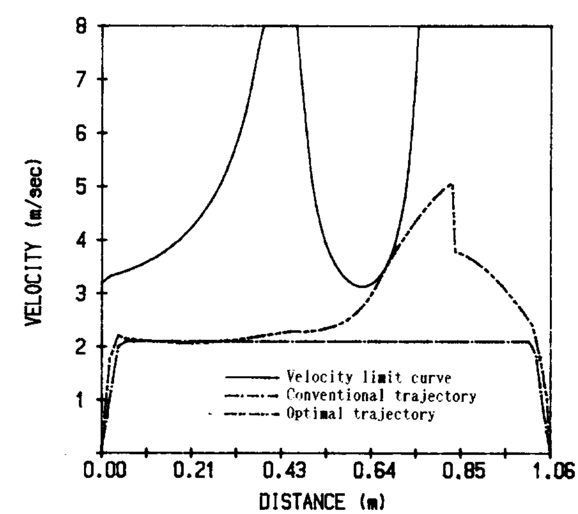

Fig. 8 Velocity limit curve and optimal velocity curve along path

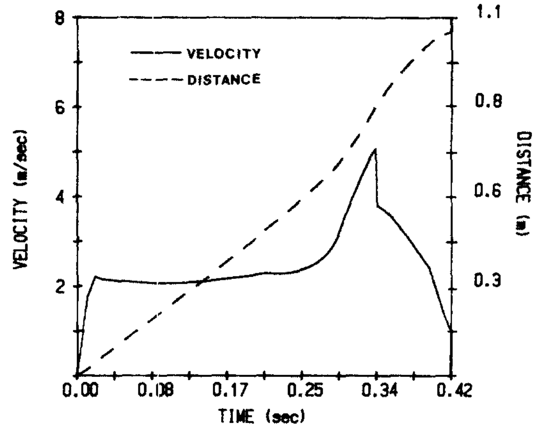

Fig. 9 Position and velocity of the end effector along path

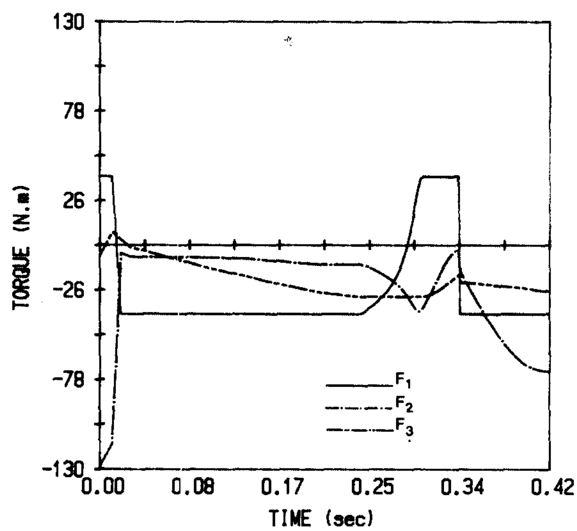

Fig. 10 Torque variation at each joint according to optimal velocity planning

The above results were compared with an operation along the straight line trajectory which directly connects the initial and final points in space. Total length of the trajectory was 894 mm which is shorter than the optimal trajectory by 310mm, but the time taken for this trajectory by applying the same minimum time planning was 0.504 sec. which is 0.085 sec. longer than the optimal treajectory.

This result indicates that there exists a trajectory for the end effector movement, which takes less time than the straight line does, and breaks our general common sense to consider the straight line as a shortest distance.

8. CONCLUSION

(1) The application of the minimum time trajectory planning reduced the operating time of the industrial manipulator by 23% compared to the conventional constant velocity or acceleration control.

(2) The straight line connecting between two points in plane or space has been considered as the shortest distance taking minimum time.
However, it was validated in this work that curved trajectory which takes less operating time than the straight line exists.

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